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An Exceptional SSM from E_6 Orbifold GUTs with intermediate LR symmetry

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ABSTRACT: We propose a class of E_6-based local orbifold Grand Unified Theories (GUTs) which yield an exceptional supersymmetric standard model as their low energy theory including leptoquark and un-Higgs exotics and a Z' at the TeV scale. Unification is achieved in two steps through an intermediate scale symmetry breaking at a unified coupling which is enhanced due to the contributions of leptoquarks to the QCD beta function.

KEYWORDS: Higher Dimensions, E6, GUT, Orbifold, Leptoquark, Multi-Component Dark Matter.

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1. Introduction

The attractiveness of supersymmetry in contemporary particle physics model building is based to some extent on the observation that in the minimal (MSSM) and next-to-minimal supersymmetric standard model (NMSSM) the coupling constants of the electroweak and strong interactions unify around 10^{16} GeV. Beyond that scale, all interactions apart from gravity might be unified into one fundamental force, mathematically modeled by a simple gauge group. Unfortunately there is no simple gauge group providing a set of irreducible representations which exclusively contain the (N)MSSM particle content. Therefore one has to either argue why the additional ("exotic") particles completing the GUT representations become heavy ("doublet-triplet splitting") [1], or take into account their effect on the running of the gauge couplings if they remain in the spectrum below the unification scale.

In the latter approach, the gauge couplings do not unify as in the (N)MSSM. Unification can be restored by introducing light incomplete GUT representations [2] or postulating an intermediate gauge symmetry[4]. In [3] it has been pointed out that unification of the gauge couplings can be achieved even in the presence of complete **27** E_6 matter if the interactions above $\Lambda_{\text{int}} \sim 10^{15}$ can be described by an $G_{PS} \times U(1)_\chi \equiv SU(4) \times SU(2)^2 \times U(1)_\chi$ gauge symmetry. As successful this approach is in the gauge coupling sector, it fails to yield a viable low energy theory:

Since in each generation, the E_6 scenario unifies matter, Higgs fields and exotics in the fundamental **27** representation, there is only one singlet in the renormalizable superpotential,

$$\mathcal{W} \sim \mathbf{27}\,\mathbf{27}\,\mathbf{27} \tag{1.1}$$

which does not discriminate between NMSSM type terms

$$\mathcal{W}_{NMSSM} \sim SHH + SD^cD + HLl^c + HQq^c \tag{1.2}$$

and leptoquark- and diquark-like couplings

$$\mathcal{W}_{LDQ} \sim Dq^cl^c + D^cQL + DQQ + D^cq^cq^c \tag{1.3}$$

which lead to fast proton decay if the triplet Higgs like exotics D_i, D_i^c have masses significantly below the GUT scale. Furthermore, as there are three copies of an NMSSM-like Higgs sector, the postulate of an H-Parity [4] allowing only one Higgs generation to couple to matter is very effective in explaining the absence of large flavor changing neutral currents (FCNCs), but inconsistent with the E_6-symmetric superpotential (1.1).

[4] The breaking scale of this symmetry could ideally be linked to a Majorana mass-term for the right-handed neutrino, triggering a see-saw mechanism and hence naturally generating small neutrino masses.

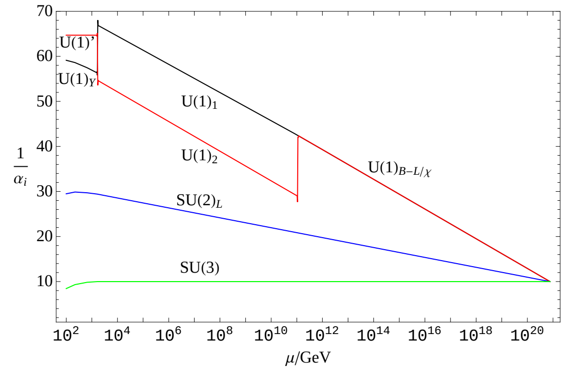

Figure 1: The unification scenario featuring three full **27** multiplets of E_6 and an intermediate $G_{LR} \times U(1)_\chi$ symmetry which is broken at the scale Λ_{int}. The gauge couplings do not unify to E_6 below the Planck scale.

If E_6 is broken to $SU(4) \times SU(2)^2 \times U(1)_\chi$ close to the Planck scale, the lepto/diquark couplings become a separate singlet and can in principle be strongly suppressed in the low energy theory. This could be achieved if the renormalizable superpotential vanished in the high energy theory and is only generated from higher dimensional operators in the course of the E_6 breaking [5], or if matter resides on a symmetry-reduced fixed point of an orbifold. In the case of intermediate Pati-Salam symmetry which unifies quarks and leptons in **4** and $\overline{\mathbf{4}}$ representations, the intermediate breaking scale is generically several orders of magnitude below the Planck scale [6],

$$\Lambda_{PS} \ll \Lambda_{E6} \sim \Lambda_{Pl} \tag{1.4}$$

predicting that the exotics D, D^c will neither have large leptoquark- nor diquark-like couplings. Thus, they behave like heavy, relatively long lived R-odd right-handed quarks in the low energy theory. This can only be relaxed if the intermediate theory does not leave $SU(4)$ intact too far below the Planck scale, which makes it more unnatural to associate Λ_{int} with the see-saw scale.

The latter objections could be resolved by reducing the degree of symmetry in the intermediate regime to a minimal left-right symmetric (N)MSSM with

$$G_{LR} \times U(1)_\chi = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_\chi \tag{1.5}$$

gauge symmetry. However, as illustrated in Fig. 1, in that case the coupling constants unify only far above the Planck scale.

The problems mentioned above seem to rule out simple E_6-invariant GUT setups in the absence of doublet-triplet splitting in the spectrum, and therefore lead us to consider higher dimensional orbifold constructions. Orbifold breaking has been proposed in [7, 8, 9] for string models. The breaking of exceptional groups in extra dimensions has been of considerable interest in recent years in the context of stringy

constructions [10, 11, 12, 13, 14, 15, 16] as well as field theoretic models [17, 18, 19, 22], most of which aim to reproduce an MSSM-like low energy theory to achieve standard unification.

From the point of view of quantum field theory, the low (or intermediate) scale gauge symmetry which we want to realize is reached in the local GUT ansatz by projecting out parts of the E_6 roots by point transformations in the extra dimensions. The corresponding gauge bosons become massive by a geometric Higgs mechanism. The fixed points of the orbifold support reduced 4D gauge theories, corresponding smaller representations and more general superpotentials, while the bulk of the extra dimensional volume retains the full gauge invariance. This construction ameliorates the severe constraints on the low-energy theory and allows us to obtain an Exceptional SSM scenario in which the effective μ term is generated by an MSSM singlet vev $\langle S \rangle$ as in the NMSSM. Furthermore, our modified unification scheme allows the full E_6 matter spectrum in the 4D theory and thus naturally yields an anomaly-free extra $U(1)$ factor. This generates the abovementioned singlet vev via its D term, thus circumventing the cosmological problems of domain walls which can plague NMSSM models with an S^3 superpotential term.

2. An Exceptional SSM with intermediate PS Symmetry

Before we turn to exceptional unification based on a $G_{LR} \times U(1)_\chi$ intermediate symmetry which requires orbifold geometries in $D = 6$ to achieve the necessary degree of symmetry breaking, we discuss possibilies and limitations of models with a $PS \times U(1)_\chi$ intermediate symmetry from 5D orbifolds.

2.1 $E_6 \to PS \times U(1)$ Breaking on $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$

Technical details about the $S^1/\mathbb{Z}_2$ orbifold construction can be found in Appendix A.2. The orbifold reflections r and r' can be endowed with order two gauge shifts to break E_6 to local $SO(10) \times U(1)$ or $SU(6) \times SU(2)$ invariance on the fixed points. We can thus break E_6 down to $PS \times U(1)_\chi$ by breaking to $SU(6) \times SU(2)_L$ and $SU(6) \times SU(2)_R$, or, more interesting for our purpose, to $SU(6) \times SU(2)_{L/R}$ and $SO(10) \times U(1)_\chi$ with $\mathbb{Z}_2$ and $\mathbb{Z}_2'$ respectively. If matter is localized on a $SO(10) \times U(1)_\chi$ fixed point, we have naturally light Higgs triplets in the **10**, and we obtain leptoquark- and diquark-like couplings of matter to the Higgs sector from the **10 16 16** superpotential term. Similarly in the $SU(6)$ case, the exotics are in the $\overline{\mathbf{15}}$ together with left- or right-handed matter and the singlet, and lepto- and diquark couplings are contained in both superpotential terms $\mathbf{6}\,\mathbf{6}\,\overline{\mathbf{15}}$ and $\overline{\mathbf{15}}\,\overline{\mathbf{15}}\,\overline{\mathbf{15}}$. This means that in this scenario it is not possible to realize exceptional unification with light exotics using brane localized matter, because lepto-diquark interactions are not suppressed. However, one can attempt to put the third generation which contains the MSSM Higgs doublet,

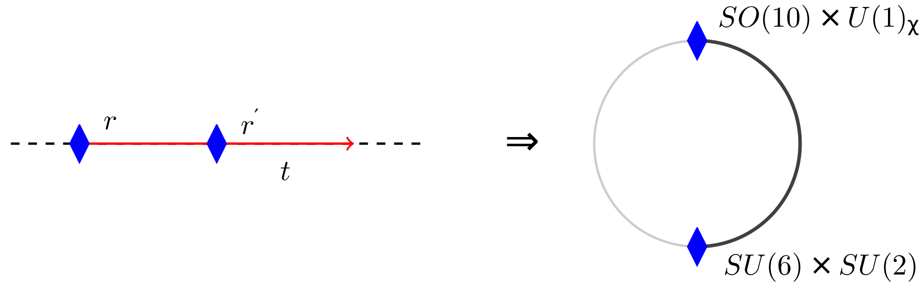

Figure 2: An $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orbifold breaking of $E_6 \to PS \times U(1)_\chi$. The two order 2 reflections are shown as blue diamonds, the translation which compactifies $\mathbb{R} \to S^1$ is shown as a red arrow.

the NMSSM-like singlet and color charged exotics, in the 5D Bulk to avoid lepto-diquark couplings while allowing quark masses at the same time. This would be a kind of orbifold-based doublet-triplet splitting, yet with what looks like complete E_6 multiplets below the compactification scale enforced by anomaly constraints. The $SU(6)\times SU(2)_{L/R}$ fixed point should only contain vectorlike matter such as the Higgs sector connected to the intermediate breaking. We now consider the $SU(6)\times SU(2)_L$ case, the case $SU(6) \times SU(2)_R$ is completely analogous except for the form of the allowed brane superpotentials.

2.2 Brane and Bulk Matter

Chiral components of the 5D gauge hypermultiplets and matter hypermultiplets can induce brane localized anomalies even though they are not in the massless spectrum. The 5D gauge multiplet can be written as a vector- and a chiral superfield and thus contains the component fields $\hat{V} \ni A_\mu, \lambda_1$ and $\hat{\chi} \ni \Sigma, \lambda_2$ where λ_i are 4D Weyl spinors and Σ is a complex scalar containing A_5 and an additional real scalar. In a $\mathbb{Z}_2$ orbifold, each 4D gauge superfield $\hat{V}^\alpha$ corresponding to a root α which is projected out entails a chiral superfield $\hat{\chi}^\alpha$ which survives on this fixed point. Since the adjoint of E_6 and of the unbroken subgroup are vectorlike representations, the surviving chiral modes also come in real representations, namely $\mathbf{16_{-3/2}} + \overline{\mathbf{16}}_{\mathbf{3/2}}$ on the $SO(10) \times U(1)_\chi$ fixed point, and $(\mathbf{20}, \mathbf{2})$ on the $SU(6) \times SU(2)$ fixed point. The 16 chiral zero modes which survive on both fixed points therefore correspond to a vectorlike representation as well.

Now, let us consider whether one can obtain full anomaly free generations from $\mathbf{27}$ representations in the bulk. 5D matter hypermultiplets consist of two oppositely charged chiral superfields $\hat{\Phi} \ni \psi, \phi$ and $\hat{\Phi}_c \ni \psi_c, \phi_c$, which together contain a 5D dirac fermion and corresponding scalar partners. In a $\mathbb{Z}_2$ orbifold, the two receive opposite boundary conditions, meaning that if $\hat{\Phi}$ is projected out on a fixed point, $\hat{\Phi}_c$ survives and vice versa. The chiral part of the hypermultiplet transforms under

$\mathbb{Z}_2$ and $\mathbb{Z}_2'$ projections as

$$\begin{aligned}\hat{\Phi}^\mu &\xrightarrow{\mathbb{Z}_2} \sigma \exp[2\pi i V \ \cdot \mu]\hat{\Phi}^\mu \\ \hat{\Phi}^\mu &\xrightarrow{\mathbb{Z}_2'} \sigma' \exp[2\pi i V' \cdot \mu]\hat{\Phi}^\mu \end{aligned} \tag{2.1}$$

if μ is the E_6 weight of the hypermultiplet, V and V' are the gauge shifts associated with $\mathbb{Z}_2$ and $\mathbb{Z}_2'$ and the parities are $\sigma, \sigma' \in \{+,-\}$. The antichiral part $\hat{\overline{\Phi}}_c$ has opposite parities. To obtain a complete light $\mathbf{27}$ from bulk hypermultiplets, one needs four $\mathbf{27}$s with parities $(\sigma,\sigma') = (+,+),(-,-),(-,+),(+,-)$, yielding chiral zero modes $(\mathbf{6},\mathbf{1},\mathbf{1})_{\mathbf{-1}} + (\mathbf{1},\mathbf{1},\mathbf{1})_{\mathbf{2}}$, $(\mathbf{4},\mathbf{2},\mathbf{1})_{\frac{\mathbf{1}}{\mathbf{2}}}$, $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{\frac{\mathbf{1}}{\mathbf{2}}}$ and $(\mathbf{1},\mathbf{2},\mathbf{2})_{\mathbf{-1}}$ respectively with our choice of gauge shifts,

$$\overline{V} = (\frac{1}{2},\frac{1}{2},0,\frac{1}{2},\frac{1}{2},0), \quad \overline{V}' = (\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},0). \tag{2.2}$$

However, the corresponding zero modes from the antichiral multiplets also form a complete $\mathbf{27}$, such that the complete matter content is vectorlike.

The possibility remains to obtain part of a $\mathbf{27}$ generation, such as the Higgs sector, from the bulk, while the remaining chiral components are localized on the $SO(10)\times U(1)_\chi$ fixed point. We are now going to implement what corresponds to a $\mathbf{10_{-1}} + \mathbf{1_2} \in \mathbf{27}$ as the zero modes of two bulk hypermultiplets in the $\mathbf{27}$. We call the two hypermultiplets $\mathbf{27^A}$ and $\mathbf{27^B}$ and give them $\mathbb{Z}_2$ parities $(+,+)^{(A)},(+,-)^{(B)}$. This produces massless chiral multiplets corresponding to

$$(\mathbf{6},\mathbf{1},\mathbf{1})^{\mathbf{A}}_{\mathbf{-1}} + (\mathbf{1},\mathbf{1},\mathbf{1})^{\mathbf{A}}_{\mathbf{2}} + (\mathbf{1},\mathbf{2},\mathbf{2})^{\mathbf{B}}_{\mathbf{-1}} \sim \mathbf{10_{-1}} + \mathbf{1_2}.$$

The superscript denotes the $\mathbf{27}$ from which the representations originate. In particular, these fields contribute the MSSM Higgs doublet and the NMSSM singlet accompanied by a generation of color charged exotics. The antichiral parts of the bulk hypermultiplets which we now call $\mathbf{27^{Ac}}$ and $\mathbf{27^{Bc}}$, also contain zero modes. $\mathbf{27^{Ac}}$ contributes extra $SU(2)_L$ charged matter, while $\mathbf{27^{Bc}}$ contributes extra $SU(2)_R$ charged matter, corresponding to representations

$$(\mathbf{4},\mathbf{2},\mathbf{1})^{\mathbf{Ac}}_{\frac{\mathbf{1}}{\mathbf{2}}} + (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})^{\mathbf{Bc}}_{\frac{\mathbf{1}}{\mathbf{2}}} \sim \mathbf{16}^{\mathbf{c}}_{\frac{\mathbf{1}}{\mathbf{2}}},$$

however with opposite 4D chirality compared to standard matter. The modes surviving on the $SU(6)\times SU(2)$ fixed point come with opposite chirality from the two hypermultiplets, and are thus anomaly free. The modes surviving on the $SO(10)\times U(1)_\chi$ fixed point are $\mathbf{10^A_{-1}}, \mathbf{1^A_2}$ and $\mathbf{10^B_{-1}}, \mathbf{1^B_2}$ in the chiral part and $\mathbf{16}^{\mathbf{Ac}}_{\frac{\mathbf{1}}{\mathbf{2}}} + \mathbf{16}^{\mathbf{Bc}}_{\frac{\mathbf{1}}{\mathbf{2}}}$ in the antichiral part. To cancel the 4D anomaly and to complete the matter content of the generation, we need brane localized chiral $\mathbf{16}'_{\frac{\mathbf{1}}{\mathbf{2}}} + \mathbf{16}^{\mathbf{3}}_{\frac{\mathbf{1}}{\mathbf{2}}}$, where one linear combination gets a mass with the antichiral bulk zero mode $\mathbf{16}^{\mathbf{c}}_{\frac{\mathbf{1}}{\mathbf{2}}}$, and the other remains in the light spectrum, e.g. as the third generation matter. The brane anomaly on

the $SO(10)\times U(1)_\chi$ fixed point, written in terms of chiral fields, now receives contributions from the two localized chiral modes $\mathbf{16'_{\frac{1}{2}}} + \mathbf{16^3_{\frac{1}{2}}}$ and bulk contributions from $2\times\mathbf{10_{-1}}+2\times\mathbf{1_2}$ and $2\times\overline{\mathbf{16}}_{-\frac{1}{2}}$. These contributions cancel since the bulk modes only contribute half of the anomaly on each of the two branes.

Such a general setup could be used to implement the ESSM models with Pati-Salam intermediate symmetries [2, 3]. Note that the couplings of the light exotics to matter, which unify lepto- and diquark terms due to the intermediate Pati-Salam symmetry, must be suppressed to avoid rapid proton decay, but should not be exactly zero as this would render the color charged exotics stable.

2.3 Superpotential

The abovementioned setup allows in principle to introduce all superpotential terms necessary for the ESSM on the branes while avoiding lepto-diquark couplings and unhiggs-matter couplings. Let $\mathbf{16^i}, \mathbf{10^i}, \mathbf{1^i}$ with $i = 1, 2$ be the first two generations of matter, unhiggs and unsinglet fields localized as chiral multiplets on the $SO(10)\times U(1)_\chi$ fixed point. The third generation is implemented as explained in the last section, and uppercase superscripts $\mathbf{A}, \mathbf{B}$ denote the two bulk hypermultiplets. Then, the following brane localized terms can be introduced. Matter receives masses from $\langle\mathbf{10^B}\rangle\mathbf{16^a16^b}$ where $a, b = 1..3$. Color charged and unhiggs exotics receive masses from $\langle\mathbf{1^A}\rangle\mathbf{10^a10^b}$ where $a, b = 1, 2, A$. Unsinglet-unhiggs mixing is generated by $\mathbf{1^j10^i}\langle\mathbf{10^B}\rangle$ where $i, j = 1, 2$. The μ term contribution can arise on both branes, namely through $\langle\mathbf{1^A}\rangle\mathbf{10^B10^B}$ on the $SO(10)\times U(1)_\chi$ fixed point, and through $\langle(\overline{\mathbf{15}},\mathbf{1})^\mathbf{A}\rangle(\mathbf{6},\mathbf{2})^\mathbf{B}(\mathbf{6},\mathbf{2})^\mathbf{B}$ on the $SU(6)\times SU(2)$ fixed point. Lepto-diquark interactions are contained in a separate singlet, $\mathbf{10^A16^a16^b}$ where $a, b, = 1..3$. While the corresponding coupling can assumed to be small, it would be interesting to investigate how its suppression can be enforced, and which bounds result for proton and exotic decay lifetimes.

This type of construction seems to exhaust the possibilities in $D = 5$ apart from introducing additional breaking from brane or bulk vevs, and higher dimensional operators. In particular, in order to obtain smaller intermediate rank 6 gauge symmetry such as $G_{LR}\times U(1)_\chi$, or fixed points with smaller gauge groups which for example allow for light leptoquark exotics and more freedom in the matter yukawa couplings, we need to study E_6 breaking in higher dimensions.

3. An Exceptional SSM with intermediate LR Symmetry

3.1 $E_6 \to G_{LR}\times U(1)$ breaking on $T^2/\mathbb{Z}_6$

We now want to turn to intermediate G_{LR} symmetry including the possibility to assign H parities and still have leptoquarks at low energies. To reach this goal purely by orbifold breaking, we require a 6D setup. The technicalities of the various relevant

$SU(6)\times SU(2)_L$ \ $SO(10)_{Q\chi}$	$\mathbf{16}_{\frac{1}{2}}$	$\mathbf{10}_{-1}$	$\mathbf{1}_{2}$
$(\overline{\mathbf{15}},\mathbf{1})$	$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$(\mathbf{6},\mathbf{1},\mathbf{1})_{-1}$	$(\mathbf{1},\mathbf{1},\mathbf{1})_{\mathbf{2}}$
$(\mathbf{6},\mathbf{2})$	$(\mathbf{4},\mathbf{2},\mathbf{1})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{2},\mathbf{2})_{-1}$	$\times$

Table 1: The decomposition of a **27** of E_6 for the rank 6 subgroups $SO(10)\times U(1)_\chi$, $SU(6)\times SU(2)_L$ and their intersection $PS\times U(1)_\chi = SU(4)_C\times SU(2)_L\times SU(2)_R\times U(1)_\chi$ which is shown inside the table. The subscript denotes $\sqrt{6}Q_\chi$, where the $U(1)$ generator is $SU(N)$ normalized. The case with $SU(6)\times SU(2)_R$ is completely analogous.

constructions based on torus compactifications are summarized in the Appendix A.3. The following model is based on a $T^2/\mathbb{Z}_6$ orbifold in 6D with E_6 gauge invariance in the bulk. $\mathbb{Z}_6$ geometries have been considered in the context of $E_8\times E_8$ heterotic string compactifications. In this sense, 6D E_6 orbifold models can be thought of as an intermediate stage from the full string theory construction to a realistic 4D model. The orbifold geometry and possible phases are specified in Section A.3.4. The gauge embeddings $G(r_3)$, $G(r_2)$ can be chosen independently, and the induced $\mathbb{Z}_6$ gauge shift is $G(r_6) = G(r_3)^{-1}G(r_2)$. This means that we can assign shift embeddings $G(r_2) : E_6 \to SO(10)\times U(1)_\chi$, and $G(r_3) : E_6 \to SU(3)^3$, which will result in $G(r_6) : E_6 \to G_{LR}\times U(1)_\chi$. This is for example achieved with the shift vector

$$\overline{V}(r_6) = (\frac{1}{6}, -\frac{1}{6}, -\frac{1}{3}, -\frac{1}{2}, -\frac{1}{6}, 0) \tag{3.1}$$

This is equivalent to a shift in the $\overline{Q}_{B-L}$ direction, appropriately normalized to be compatible with the $\mathbb{Z}_6$ algebra. No discrete Wilson line is allowed. This gives us three inequivalent fixed points with localized gauge invariances $G_{LR}\times U(1)_\chi$, $SO(10)\times U(1)_\chi$ and trinification $SU(3)^3$. This can be understood from Table 7 since those groups correspond to roots with integer, one-third integer and half integer charge under Q_{B-L}, respectively. The construction is shown in Figure 3.

3.2 Bulk Matter and Anomalies on the $\mathbb{R}^2/\mathbf{632}$ orbifold

In the SUSY version of the model, vectors are simply extended by massless Weyl gaugino modes in the adjoint, and thus give us standard 4D $\mathcal{N}=1$ multiplets corresponding to the unbroken gauge group. However, the scalar zero modes usually do not appear in vectorlike pairs and therefore result in massless chiral multiplets in complex representations satisfying $V\cdot\alpha\in\mathbb{Z}+1/6$ where V is the $\mathbb{Z}_6$ gauge shift. In the case of $E_6\to G_{LR}\times U(1)_\chi$ breaking on the $\mathbb{R}^2/\mathbf{632}$ orbifold, they contain quark-like exotics with $Q_\chi=\pm 3/2$, using the notation from Table 7 they are Q_{16} from $\overline{\mathbf{16}}_{3/2}$ and U_{16}, D_{16} from $\mathbf{16}_{-3/2}$ or vice versa, depending on the choice of the gauge shift. Note that they do not form complete $SU(3)^3$ or $SO(10)\times U(1)_\chi$ representations but

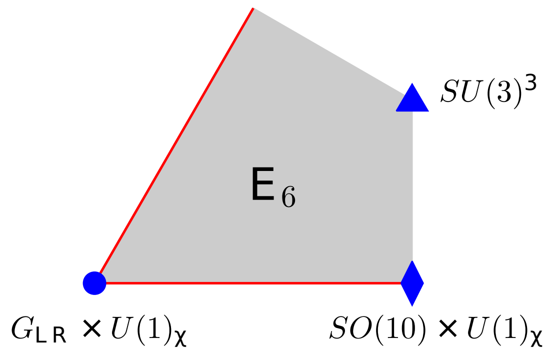

Figure 3: An $E_6 \to G_{LR} \times U(1)_\chi$ breaking scenario on the $\mathbb{R}^2/\mathbf{632}$ orbifold and the local gauge groups at the $\mathbb{Z}_6$, $\mathbb{Z}_3$ and $\mathbb{Z}_2$ fixed points which are shown as blue circle, triangle and diamond. The shaded area shown is the fundamental domain only, and long (red) and short sides are identified. The construction is outlined in Section A.3.4.

appear in G_{LR} representations, nor are they by themselves anomaly free with respect to the unbroken gauge group $G_{LR} \times U(1)_\chi$. Since we preserve our extra $U(1)_\chi$ or $U(1)'$ at low energies, these exotics do not mix with standard matter from the **27**. However, they induce 4D anomalies and localized anomalies on the $\mathbb{Z}_6$ and $\mathbb{Z}_3$ fixed points, where the chiral bulk fermions from the gauge superfield $\hat{\chi}$ which would survive the individual orbifold projections on one fixed point are in $(\mathbf{3}, \mathbf{2}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{3}, \mathbf{3}, \overline{\mathbf{3}})$ representations of $G_{LR} \times U(1)_\chi$ and $SU(3)^3$ respectively, while on the $\mathbb{Z}_2$ fixed point, vectorlike contributions corresponding to $\mathbf{16}_{-3/2} + \overline{\mathbf{16}}_{3/2}$ of $SO(10) \times U(1)_\chi$ survive the projection and cancel. To cancel the 4D anomalies, one can put the completion consisting of a $(\overline{\mathbf{3}}, \mathbf{2}, \mathbf{1}) + (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{2})$ on the $\mathbb{Z}_6$ fixed point, which leaves open the problem how the 6D brane localized anomalies are cancelled. A simpler way to cancel the 4D and brane anomalies while removing the chiral bulk modes is the doubling of the bulk fermion content with an additional hypermultiplet in the adjoint **78**. The hypermultiplet has opposite 6D chirality compared to the 6D gauginos, so the fermions of both form an unconstrained 6D spinor. The $\mathbb{Z}_6$ parity of the hypermultiplet must be chosen such that the surviving 4D chiral modes from $\hat{\Phi}$ on each fixed point are in the same representation as the 4D antichiral zero modes from $\hat{\chi}$. Under a $\mathbb{Z}_6$ rotation, the vector hypermultiplet belonging to a root α transforms as

$$\begin{aligned} \hat{V}^\alpha &\longrightarrow e^{2\pi i V\cdot\alpha} \hat{V}^\alpha \\ \hat{\chi}^{-\alpha} &\longrightarrow e^{-2\pi i V\cdot\alpha} e^{-\frac{2\pi i}{6}} \hat{\chi}^{-\alpha}. \end{aligned} \tag{3.2}$$

The chiral superfields in an adjoint hypermultiplet belonging to a root α transform as

$$\hat{\Phi}^\alpha \longrightarrow e^{2\pi i V\cdot\alpha} e^{-\frac{2\pi i}{6} a_+} \hat{\Phi}^\alpha$$

$$\hat{\Phi}_c^{-\alpha} \longrightarrow e^{-2\pi i V\cdot\alpha} e^{-\frac{2\pi i}{6}a_-} \hat{\Phi}_c^{-\alpha} \tag{3.3}$$

From invariance of the 6D hypermultiplet kinetic term [26], $\int d^2\theta\, \hat{\Phi}_c^{-\alpha}\partial\hat{\Phi}^{\alpha}$ (where $\partial = (\partial_5 - i\partial_6)/\sqrt{2}$ transforms like $\partial \to e^{-2\pi i/6}\partial$ under orbifold rotations in analogy to the 6D gauge field $\hat{\chi}$), $a_+ + a_- \in 6\mathbb{Z} - 1$. For the choice $a_+ = 5$, $a_- = 0$, every invariant mode $\hat{\chi}^{-\alpha}$ is accompanied by an invariant mode $\hat{\Phi}^{\alpha}$, and the chiral zero modes from $\hat{\chi}$ and $\hat{\Phi}$ appear in mutually complex conjugate representations. The invariant modes of $\hat{\Phi}_c$ are in the adjoint of the unbroken gauge group, and localized and 4D gauge anomalies are cancelled and we assume all extra bulk matter to have masses at the compactification scale. This has the interesting consequence that the Kaluza-Klein excitations in the adjoint appear in 4D $\mathcal{N} = 4$ multiplets and therefore the contributions to powerlike threshold corrections to gauge unification can vanish [24]. In a full embedding in $E_8 \times E_8$ heterotic models, after breaking of $E_8^{vis} \to G_{LR} \times U(1)^3$, all but one of the $U(1)$ factors are automatically anomaly free. It remains a subject of further study how this could be realized in the context of the exceptional SSM proposed in this paper.

Analogous to the 5D model with intermediate $PS \times U(1)_\chi$ symmetry, one can consider implementing parts of a generation, for example the Higgs fields, as 6D bulk fields rather than as being purely localized on a fixed point. The orbifold transformation properties for a bulk hypermultiplet of weight μ are analogous to (3.3),

$$\begin{aligned}\hat{\Phi}^{\mu} &\longrightarrow e^{2\pi i V\cdot\mu} e^{-\frac{2\pi i}{6}a_+} \hat{\Phi}^{\mu} \\ \hat{\Phi}_c^{-\mu} &\longrightarrow e^{-2\pi i V\cdot\mu} e^{-\frac{2\pi i}{6}a_-} \hat{\Phi}_c^{-\mu}\end{aligned} \tag{3.4}$$

where again, the integers $a_+ = 0\ldots5$ and $a_- = 5 - a_+$ can be chosen for each hypermultiplet. A complete $\mathbf{10_{-1}} + \mathbf{1_2}$ from the bulk could be realized by three $\mathbf{27}$ with parities $a_+ = 0, 2, 4$. With it comes a complete massless $\overline{\mathbf{16}}_{-\frac{1}{2}}$ from the antichiral part. However, the resulting brane anomaly structure is nontrivial [20, 21]. This construction results in a vanishing anomaly on the $\mathbb{Z}_3$ fixed point, and the modes contributing to the $\mathbb{Z}_2$ and $\mathbb{Z}_6$ brane anomalies come in complete $SO(10)$ multiplets. In the conventions given in [20, 21], the representations producing the localized anomalies, both equivalent to $(\mathbf{10_{-1}} + \mathbf{1_2} + \overline{\mathbf{16}}_{-\frac{1}{2}})$ on the $\mathbb{Z}_6$ fixed point and on the $\mathbb{Z}_2$ fixed point, contribute with prefactors $\frac{1}{4}$ and $\frac{3}{4}$ respectively to the remaining 4D anomaly. Since the factors are not (half) integers, even after brane localized matter $2 \times \mathbf{16}_{\frac{1}{2}}$ is added to obtain the correct spectrum and cancel the 4D anomaly, localized $U(1)$ anomalies remain which would have to be cancelled by some mechanism. For the remainder of our discussion we therefore assume brane localized representations corresponding to complete $\mathbf{27}$ to provide the matter content.

3.3 Local Matter content and gauge unification

In accordance with the local GUT framework, the massless spectrum on orbifold

$SU(3)^3$ \ $SO(10)_{Q\chi}$	$\mathbf{16}_{\frac{1}{2}}$	$\mathbf{10}_{-1}$	$\mathbf{1}_{2}$
$\mathbf{A} = (\overline{\mathbf{3}},\mathbf{1},\mathbf{3})$	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{2})_{(-\frac{1}{3},\ \frac{1}{2})}$	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{(\ \frac{2}{3},-1)}$	$\times$
$\mathbf{B} = (\mathbf{3},\mathbf{3},\mathbf{1})$	$(\mathbf{3},\mathbf{2},\mathbf{1})_{(\ \frac{1}{3},\ \frac{1}{2})}$	$(\mathbf{3},\mathbf{1},\mathbf{1})_{(-\frac{2}{3},-1)}$	$\times$
$\mathbf{C} = (\mathbf{1},\overline{\mathbf{3}},\overline{\mathbf{3}})$	$(\mathbf{1},\mathbf{2},\mathbf{1})_{(-1,\ \frac{1}{2})}$ $(\mathbf{1},\mathbf{1},\mathbf{2})_{(\ 1,\ \frac{1}{2})}$	$(\mathbf{1},\mathbf{2},\mathbf{2})_{(\ 0,-1)}$	$(\mathbf{1},\mathbf{1},\mathbf{1})_{(\ 0,2)}$

Table 2: The decomposition of a **27** of E_6 for the rank 6 subgroups $SO(10) \times U(1)_\chi$, $SU(3)_C \times SU(3)_L \times SU(3)_R$ and their intersection $G_{LR} \times U(1)_\chi = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_\chi$ which is shown inside the table. The subscript denotes $(2\sqrt{6}/3\, Q_{\text{B-L}},\ \sqrt{6}Q_\chi)$, where the $U(1)$ generators are $SU(N)$ normalized.

fixed points is not determined by the unbroken 4D gauge invariance, but – as it is necessary to obtain a consistent gauge theory – by the local gauge invariance on the fixed point itself, which is in general larger. This is in contrast to bulk matter the massless modes of which generally only respect the low energy gauge symmetry. The branching rules of the anomaly-free field content of a complete **27** of E_6 for the rank 6 subgroups $SO(10) \times U(1)_\chi$, $SU(3)^3$ and their intersection $G_{LR} \times U(1)_\chi$ are given in Table 2.

Since $\mathcal{N} > 1$ SUSY does not allow chiral matter, we have reduced the overall SUSY content to $\mathcal{N} = 1$. This is achieved automatically by the orbifold projection together with the choice of the embedding of $\mathbb{Z}_6$ into the R symmetry generated by I_R^3, analogous to orbifold twists in 10D $\mathbb{Z}_6$ orbifolds (cf. appendix C.2). Similarly to gauge theory, the local amount of supersymmetry can generally be larger. In the $\mathbb{Z}_6$ orbifold with 6D $\mathcal{N} = 1$ SUSY in the bulk, the amount of supersymmetry is reduced to 4D $\mathcal{N} = 1$ on all fixed points. This is not always the case in $T^6/\mathbb{Z}_6$ compactifications, for example in the $\mathbb{Z}_6$-II orbifold. We assume here that possible underlying 10D shifts reduce the amount of supersymmetry to 4D $\mathcal{N} = 1$ on all 6D fixed points. Unless further constraints are taken into account, there is no *a priori* rule how the three matter generations should be implemented using the abovementioned field content. If they are spread over the various fixed points, the superpotential must arise from effective nonlocal interactions respecting the parities of the local GUT representations. This can include scenarios where a vectorlike pair of chiral/antichiral multiplets and another chiral multiplet combine to a massive multiplet and a chiral multiplet in the light spectrum. There are some plausible choices: it is for example most appealing to implement the third generation on the fixed point with trinification symmetry where all fields have to be H-even. The localized field content there is thus $\{\mathbf{A}_3, \mathbf{B}_3, \mathbf{C}_3\}$. This setup accommodates the large top mass and has the additional advantage that the exotics D_3^c, D_3 can be true leptoquarks without diquark interactions since we can forbid the $\mathbf{A}_3^3$ and $\mathbf{B}_3^3$ terms in the superpotential by some

parity. Furthermore, there is for now only one singlet Higgs S and one generation of MSSM Higgs doublets H_u, H_d. The light generations must be localized on the other two types of fixed points since here we want to implement an H parity to prevent mixing of the un-Higgs doublets (and singlets) and exotics with standard Higgs fields and matter which would potentially produce fatal contributions to FCNCs. If a light generation is localized on the chiral $G_{LR} \times U(1)_\chi$ fixed point, its color charged exotics can be leptoquarks as well, if it resides on the vectorlike $SO(10) \times U(1)_\chi$ fixed point, they are H-odd and lepto/diquark interactions are forbidden.

We now want to use our freedom to place representations of the local gauge groups on the fixed points as boundary localized matter, such that the minimally required particle content of the (E)NMSSM is included, with the simultaneous aim of obtaining unification below the Planck scale. We have seen that the group $G_{LR} \times U(1)_\chi$ does not unify in 4D with complete E_6 multiplets. One therefore might try to work with incomplete E_6 multiplets. The minimal field content on the $SO(10) \times U(1)_\chi$ invariant fixed points, $\mathbf{16}_1 + \mathbf{16}_2$, could provide the matter content of the two light generations without further exotics. Unfortunately, the standard $U(1)_\chi$ charge assignment to matter as inherited from the $\mathbf{27}$, $Q_\chi^{\mathbf{16}} = 1/2$, gives us 4D triangle anomalies which are only cancelled by the exotics with $Q_\chi^{\mathbf{10}} = -1$ and $Q_\chi^{\mathbf{1}} = 2$ in the full multiplet. One might consider assigning a special charge $Q_\chi = -1/2$ to the first generation matter to make the incomplete matter vector-like under $U(1)_\chi$. This cancels the $SO(10) \times U(1)_\chi$ anomalies, but does not allow a superpotential at tree-level, and requires breaking $U(1)_\chi$ at a high scale to generate it. We therefore use the full three generation particle content and achieve unification below the Planck scale by different means.

To calculate the unification scenario, we need as one further ingredient the breaking mechanism of the intermediate symmetry,

$$G_{LR} \times U(1)_\chi \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'. \tag{3.5}$$

Giving some Higgs fields H_{int}, $\bar{H}_{\text{int}}$ vevs in $\tilde{\nu}^c$ and $\tilde{\bar{\nu}}^c$ directions respectively at Λ_{int} leaves arbitrary linear combinations of

$$g_Y Y = \frac{g_R g_{B-L} \left(\sqrt{3} T_R^3 + \sqrt{2} Q_{B-L}\right)}{\sqrt{3 g_{B-L}^2 + 2 g_R^2}} \qquad \text{and} \tag{3.6}$$

$$g' Q' = \frac{g_\chi \left(g_R^2 \left(2\sqrt{3} Q_\chi - \sqrt{2} T_R^3\right) + \sqrt{3} g_{B-L}^2 \left(Q_{B-L} + 3 Q_\chi\right)\right)}{\sqrt{\left(3 g_{B-L}^2 + 2 g_R^2\right)\left(9 g_{B-L}^2 + 6 g_R^2 + g_\chi^2\right)}} \tag{3.7}$$

invariant, yielding the matching conditions for the corresponding coupling constants via the GUT normalization of the charges

$$g^Q = \sqrt{2 \mathrm{tr}_a [(g^Q Q_a)^2]}. \tag{3.8}$$

Like in models with intermediate Pati-Salam symmetry, this can be accomplished by using a $\mathbf{27}+\overline{\mathbf{27}}$ which does not lead to unification as demonstrated in Fig. 1. In our case, we can consider to use parts of $\mathbf{27}+\overline{\mathbf{27}}$, namely $\mathbf{16}+\overline{\mathbf{16}}$ on the $SO(10)\times U(1)$ fixed points, an $(\mathbf{1},\overline{\mathbf{3}},\overline{\mathbf{3}})+(\mathbf{1},\mathbf{3},\mathbf{3})$ on trinification fixed points, or a representation of $G_{LR}\times U(1)_\chi$ as given in Table 2 (alternatively, one can consider putting an $(\mathbf{1},\mathbf{1},\mathbf{3})+(\mathbf{1},\mathbf{1},\overline{\mathbf{3}})$ on a trinification fixed point which is not contained in a small E_6 multiplet and has different quantum numbers). For these breaking scenarios to be viable, potentials need to be found which produce these vevs in a D-flat direction. For further analysis, from here on we assume for our unification scenario that there is no additional matter content below Λ_{int}. Two choices for the additional matter doing the job of the intermediate symmetry breaking are

$$i)\quad L, l^c, \langle\nu^c\rangle + c.c. \sim (\mathbf{1},\overline{\mathbf{3}},\overline{\mathbf{3}})\cap\mathbf{16}+c.c. \tag{3.9}$$

$$ii)\quad L, l^c, \langle\nu^c\rangle, H_u, H_d, S + c.c. \sim (\mathbf{1},\overline{\mathbf{3}},\overline{\mathbf{3}})+c.c. \tag{3.10}$$

These representations affect the running between Λ_{int} and Λ_{E6}, and would correspond to putting the intermediate Higgs on a $G_{LR}\times U(1)_\chi$ invariant fixed point or the more symmetric trinification localized case, respectively. Since we are free to split heavy vector-like $\mathbf{27}+\overline{\mathbf{27}}$ at Λ_{E6}, these choices do not necessarily correspond to really incomplete multiplets. Some examples for the running with such a breaking matter content are shown in Fig. 4. These scenarios unify below (though in one case close to) the Planck scale, and since in δ extra dimensions we can roughly identify the extra dimensional volume with $V_\delta\sim\Lambda_{E6}^{-\delta}$, they also unify below the fundamental gravity scale M_* because of $M_*/\Lambda_{E6}\sim(\Lambda_{Pl}/\Lambda_{E6})^{2/(2+\delta)}$. The unified coupling at the E_6 scale is $\alpha(\Lambda_{E6})\approx\alpha_s(m_Z)\approx 0.1$. Those scenarios with larger intermediate particle content tend to unify at a higher E_6 scale and to have a lower intermediate breaking scale Λ_{int}. The exact E_6 unification which we assume here for simplicity is in general modified by threshold corrections from the local GUT compactification, which results in a modified intermediate breaking scale. The analysis of the impact of these corrections to unification and low energy observables is beyond the scope of this work, but must be taken into account in order to precisely determine the breaking scenario from measurements of couplings at the TeV scale. Below Λ_{int}, the right-handed neutrino is integrated out, leaving a matter content in incomplete $\mathbf{27}$, which results in a mixing of the two Abelian gauge groups $U(1)_Y$ and $U(1)'$ at the 1-loop level. However, there is a basis $\{U(1)_1, U(1)_2\}$ in which the two couplings run independently. At the scale, where the NMSSM-like singlet S acquires a vev, $U(1)_Y$ is projected out as the unbroken linear combination of $U(1)_1$ and $U(1)_2$. The broken generator corresponds to $U(1)'$, resulting in a heavy Z' boson with mass of around a TeV. Its coupling to matter is roughly of the strength of the SM hypercharge at the Z pole. A list containing the numerical values of the $U(1)'$ couplings and charges, corresponding to the four scenarios presented in Fig. 4, can be found in Table 3. It lists the charges and coupling strengths of the matter coupling to the Z' boson. At

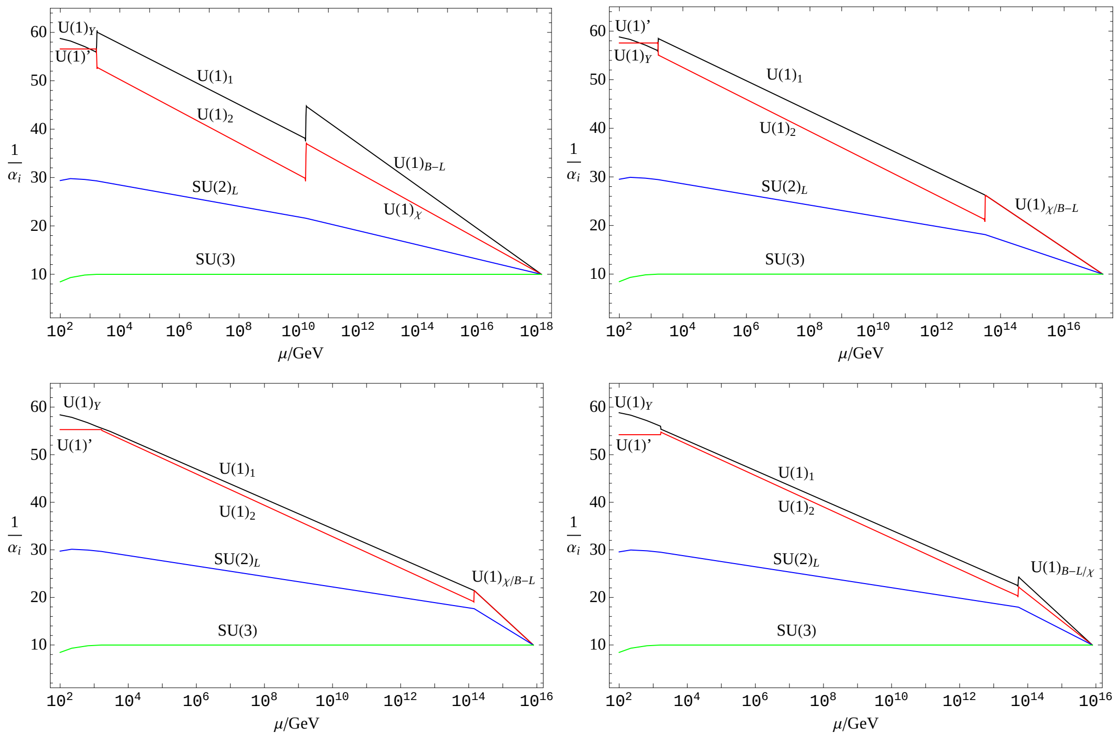

Figure 4: Four unification scenarios at 1-loop with matter in complete E_6 multiplets and intermediate breaking with combinations of incomplete vector-like Higgs representations $i)$ and $ii)$ as allowed by the orbifold compactification. The cases are $i), ii), 3 \times ii)$ and $2 \times ii) + 1 \times i)$. Below the intermediate scale, the mixing of the Abelian gauge groups is taken into account.

this stage, threshold corrections from orbifold effects are not yet taken into account. As Q' depends on the ratio of the gauge couplings at Λ_{int}, the charges are generally non-rational.

3.4 Low-energy scenario and phenomenological aspects

Decomposing the superpotential from (A.15) under SM $\times U(1)'$, one obtains:

$$\begin{aligned} W \;=\; & Y^u\, u^c\, Q\, H^u \,+\, Y^d\, d^c\, Q\, H^d \,+\, Y^e\, e^c\, L\, H^d \\ & + Y^D\, D\, u^c\, e^c \,+\, Y^{D^c}\, D^c\, L\, Q \,+\, Y^{SD}\, S\, D^c\, D \,+\, Y^{SH}\, S\, H^u\, H^d\,. \end{aligned} \tag{3.11}$$

Note, that this already incorporates R-parity since $U(1)_{\text{B-L}}$ is contained in E_6. In general, for example if the corresponding matter originates from fixed points where E_6 is broken to $H \subset SO(10) \times U(1)_\chi$, the trinification relations between Y^{SD}, Y^{SH} and Y^e, and between Y^u, Y^d,Y^D, Y^{Dc} are relaxed, which allows the superpotential to respect H parity. If at least one generation of color charged exotics D, D^c have leptoquark couplings (and are thus H-even) as it is allowed by scenarios with trinification

$H_{\rm int}, \bar{H}_{\rm int}$	$i)$	$ii)$	$3ii)$	$i) + 2ii)$
$\Lambda_{\rm int}/{\rm GeV}$	1.6×10^{10}	3.0×10^{13}	1.3×10^{14}	4.9×10^{13}
$\Lambda_{\rm GUT}/{\rm GeV}$	1.3×10^{18}	1.5×10^{17}	7.2×10^{15}	7.2×10^{15}
$g'\vert_{M_{Z'}}$	0.471	0.467	0.476	0.482
Q'_X				
Q	0.224	0.231	0.234	0.232
u^c	0.283	0.261	0.250	0.257
d^c	0.055	0.067	0.073	0.069
D	-0.449	-0.462	-0.468	-0.464
D^c	-0.339	-0.328	-0.322	-0.326
L	0.114	0.097	0.089	0.094
e^c	0.165	0.201	0.218	0.208
H^u	-0.508	-0.492	-0.484	-0.489
H^d	-0.279	-0.298	-0.307	-0.301
S	0.787	0.790	0.790	0.790

Table 3: $U(1)'$ charges and couplings corresponding to the four scenarios from Fig. 5. $i)$ and $ii)$ label the matter content as in eq. (3.9) which breaks the intermediate gauge group.

or $G_{LR} \times U(1)_\chi$ intermediate breaking, it is conceivable that the H-odd exotics can decay into H-odd singlets and H-even leptoquarks. This might circumvent the need of models with intermediate $PS \times U(1)_\chi$ and no leptoquarks for a small H parity breaking if such a scenario can be brought to be consistent with cosmological bounds on neutral dark matter.

Electroweak symmetry breaking is triggered by the S field obtaining a vev which appears to be naturally induced via radiative effects, as its coupling to the exotics drives the soft-breaking mass parameter m_S^2 to negative values. The quartic term in S originates from the $U(1)'$ D term, linking the vev of the singlet to the mass of the Z' boson. In order to be consistent with experimental data, the singlet vev has to be above one TeV, generically decoupling the MSSM-like Higgs sector from the singlet field.

4. Conclusions and Outlook

We have demonstrated how $\mathbb{Z}_n$ orbifold constructions can be used to generate an E_6 inspired Exceptional SSM with a D term induced singlet vev, circumventing many problems of comparable standard E_6 GUT constructions. While models with intermediate $PS \times U(1)_\chi$ symmetry can be constructed in 5D orbifolds, the aim to have intermediate LR symmetry led us to consider 6D geometries. The resulting models potentially provide a rich phenomenology at collider experiments: In addition to

the generic R-parity conserving supersymmetric signatures, a Z' boson with a mass around the TeV scale as well as strongly produced leptoquark-like exotics should guarantee a high discovery potential at the LHC. This is in contrast to $SU(5)$ derived standard GUT unification scenarios which offer no directly observable collider phenomenology beyond the MSSM.

Apart from R-parity, there is H-parity rendering the lightest un-Higgs or un-Higgsino stable (LHP). If the lightest particle which is odd under both parities is not heavy enough to decay into lighter singly-odd particles, there may even be a third type of dark matter. As there are multiple ways from the resulting multicomponent relic densities to be in agreement with current bounds from WMAP and astrophysical observations, they can add interesting aspects to the interpretation of recent direct and indirect WIMP searches.

On the theoretical side, it would be interesting to explore whether such or similar scenarios can be embedded in complete heterotic string models. Those impose strict additional constraints and thus can reduce the level of arbitrariness in the choice of Wilson lines, representations and the massless spectrum which is inherent to the effective field theory approach. In this context it should also be possible to address the generation of the effective superpotential needed if matter is localized on different fixed points. This is postulated in the field theory construction, and we have only sketched this in this paper. It is also noteworthy that the 1-loop QCD beta function vanishes due to the color charged light exotics, and the unified gauge coupling in $SU(N)$ normalization, $\alpha(\Lambda_{E6}) \approx 0.1$, is therefore considerably larger than in standard MSSM unification.

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A. LR symmetric E_6 breaking on a Circle and a Torus

A.1 Shift embeddings for $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z}_4$ and $\mathbb{Z}_6$

Since we are interested in LR symmetric intermediate groups of rank 6, we restrict the orbifold action in group space to those generated by the Cartan generators. Let θ be a generator of an orbifold symmetry, e.g. rotations, translations or parities. We can now associate with it an action in group space which we parameterize with a

shift vector V acting on the roots of E_6. The roots thus transform as

$$G(\theta)E_\alpha = \exp\left[2\pi i V \cdot H\right] E_\alpha = \exp\left[2\pi i V \cdot \alpha\right] E_\alpha \tag{A.1}$$

where we demand that V is chosen to be compatible with the multiplication law of the orbifold space group, i.e.

$$G(\theta)^n = \exp\left[2n\pi i V \cdot H\right] = 1 \tag{A.2}$$

for $\mathbb{Z}_n$. This is equivalent to

$$\forall \alpha : V \cdot \alpha \in \mathbb{Z}/n \tag{A.3}$$

and nV should thus be element of the co-root lattice. The unbroken subgroup which survives under such an orbifold action is given by the set of generators (the Cartan subalgebra and roots) which are left invariant under the action of θ.

To narrow down the number of candidates, we demand that the minimal viable LR symmetric subgroup of rank 6, $G_{LR} \times U(1)_\chi$, in the embedding given in Table 7 is contained in the invariant part $H \subset E_6$. The resulting groups and representative shift vectors are shown in Table 4. For $\theta^2 = 1$, we find the two largest subgroups. If we extend our reach to transformations which obey $\theta^3 = 1$, we find the trinification group. The case $\theta^4 = 1$ includes the $\mathbb{Z}_2$ candidates, and two new groups containing $SU(3)_{L/R}$. Combining two orbifold parities, we find the common subgroups given in Table 5. Since the generators which survive on any fixed point or line are even under θ, we have to define boundary localized matter to be even under the orbifold twist as well. Likewise, of any matter representation with weights μ_i, which transforms as (e.g. bulk matter)

$$G(\theta)|\mu_i\rangle = e^{2k\pi i/n} \exp\left[2\pi i V \cdot \mu_i\right] |\mu_i\rangle, \quad k = 1, \ldots, n \ , \tag{A.4}$$

only the even parts remain on the fixed point.

A.2 Orbifold Geometries in $D = 5$

In $D = 5$ there are essentially two orbifolds, $S^1/\mathbb{Z}_2$ and $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$, which have as fundamental domains an interval $[0, \pi R]$ with boundary conditions imposed on both ends (corresponding to the fixed points). The former can be seen as a special case of the latter without discrete Wilson line. An exemplary construction is shown in Figure 2. One can start with the compactification of the real line $\mathbb{R}$ to S^1 by modding out a translation t. An order 2 reflection about the origin r induces a second reflection $r' = tr$. The fundamental domain is an interval with the fixed points of r and r' as boundaries. One can now assign different $\mathbb{Z}_2$ gauge shifts to the two reflections r and r' to break E_6 down to subgroups on the two fixed points, resulting in a discrete Wilson line.

$\mathbb{Z}_2$	Subgroup H	Shift $2\overline{V}$
	$SO(10) \times U(1)_\chi$	$(1,1,0,1,1,0)$
	$SU(6) \times SU(2)_R$	$(0,0,1,0,0,0)$
	$SU(6) \times SU(2)_L$	$(1,1,1,1,1,0)$
$\mathbb{Z}_3$	Subgroup H	Shift $3\overline{V}$
	$SU(3)_C \times SU(3)_L \times SU(3)_R$	$(0,0,1,2,0,0)$
$\mathbb{Z}_4$	Subgroup H	Shift $4\overline{V}$
	$SU(5) \times U(1) \times SU(2)_L$	$(3,1,3,1,1,0)$
	$SU(5) \times U(1) \times SU(2)_R$	$(2,2,1,0,2,0)$
	$SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_\chi$	$(3,1,2,3,1,0)$
	$SU(3)_C \times SU(3)_L \times SU(2)_R \times U(1)$	$(0,0,1,2,0,0)$
	$SU(3)_C \times SU(3)_R \times SU(2)_L \times U(1)$	$(3,1,1,1,1,0)$
$\mathbb{Z}_6$	Subgroup H	Shift $6\overline{V}$
	$SU(3)_C \times SU(3)_L \times SU(2)_R \times U(1)$	$(4,2,1,0,2,0)$
	$SU(3)_C \times SU(3)_R \times SU(2)_L \times U(1)$	$(5,1,5,3,1,0)$
	$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_\chi$	$(1,5,4,3,5,0)$

Table 4: The subgroups $E_6 \supset H \supset G_{LR} \times U(1)_\chi$ invariant under shifts which are compatible with $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z}_4$ amd $\mathbb{Z}_6$. Representative shift vectors for each case are given in the dual basis such that $V \cdot \alpha = \overline{V} \cdot \Delta(\alpha)$.

$\mathbb{Z}_2 \times \mathbb{Z}_2$	$SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_\chi$
$\mathbb{Z}_2 \times \mathbb{Z}_3$	$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_\chi$
	$SU(3)_C \times SU(3)_L \times SU(2)_R \times U(1)$
	$SU(3)_C \times SU(3)_R \times SU(2)_L \times U(1)$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_\chi$
	$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_\chi$
$\mathbb{Z}_3 \times \mathbb{Z}_4$	$SU(3)_C \times SU(3)_L \times SU(2)_R \times U(1)$
	$SU(3)_C \times SU(3)_R \times SU(2)_L \times U(1)$
	$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_\chi$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_\chi$
	$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_\chi$

Table 5: The nontrivial ($H_i \nsubseteq H_j$) common invariant subgroups $H_i \cap H_j$ under combinations of two shifts.

A.3 Orbifold Geometries in $D = 6$

We now want to consider the possibilites how E_6 can be broken to suitable rank 6 subgroups containing $G_{LR} \times U(1)_\chi$ using two extra dimensions. In general, we can

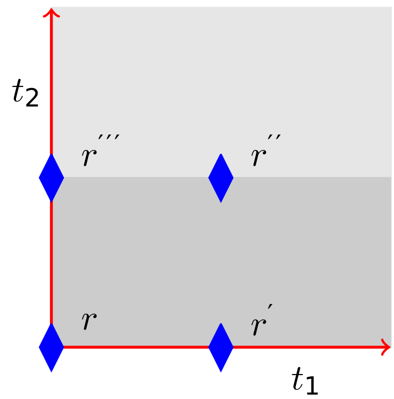

Figure 5: The $\mathbb{R}^2/\mathbf{2222}$ orbifold which is one of the $T^2/\mathbb{Z}_2$ orbifolds. The blue diamonds indicate the fixed points under the four 180° rotations, while the red arrows are the translations which span the torus. The light shaded area indicates the torus, the dark shaded area corresponds to the fundamental domain of the orbifold.

consider $\mathbb{R}^2/\Gamma$ where Γ is one of the 17 plane crystallographic groups. To obtain a gauge symmetry close to $G_{PS} \times U(1)_\chi$ or $G_{LR} \times U(1)_\chi$ and hence less constrained superpotentials, we search for 6D geometries with fixed points in which two generators, either rotations or translations, can carry nontrivial gauge embeddings. In this paper, we consider only rotational embeddings, and leave the possibilities of geometries involving mirror planes such as the orbifolds $\mathbf{3}{*}\mathbf{3}$ and ${*}\mathbf{333}$ for future investigation[5]. We discuss some examples which can be useful in LR symmetric E_6 breaking.

A.3.1 Maximal Subgroups from a $\mathbb{Z}_2$ Orbifold

The $T^2/\mathbb{Z}_2$ geometry $\mathbb{R}^2/\mathbf{2222}$ as shown in Fig. 5 is generated by modding out a torus (not necessarily a 90° one as shown here) with a 180° (order 2) rotation r about the origin, to which we can assign a shift embedding

$$G(r)E_\alpha \in \{1, -1\}E_\alpha \ . \tag{A.5}$$

Together with the two translations t_1 and t_2 denoted by the red arrows, this induces four rotations of order 2 shown as blue diamonds. They are generated by

$$r' = t_1 r, \quad r'' = t_2 t_1 r, \quad r''' = t_2 r \tag{A.6}$$

The orbifold offers the possibility to assign two $\mathbb{Z}_2$-discrete Wilson lines to t_1 and t_2. The resulting gauge shifts are

$$G(r')E_\alpha = G(t_1)G(r)E_\alpha \in \{1, -1\}E_\alpha \tag{A.7}$$

[5]For the crystallographic groups we use the orbifold notation, cf. [23].

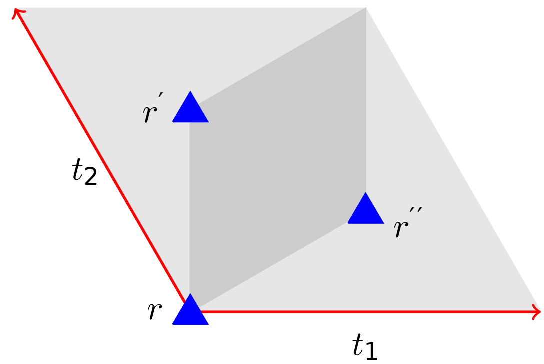

Figure 6: The $\mathbb{R}^2/\mathbf{333}$ orbifold which is one of the $T^2/\mathbb{Z}_3$ orbifolds. The blue triangles indicate the fixed points under the three 120° rotations, while the red arrows are the translations which span the torus. The light shaded area indicates the torus, the dark shaded area corresponds to the fundamental domain of the orbifold.

$$G(r'')E_\alpha = G(t_2)G(t_1)G(r)E_\alpha \in \{1,-1\}E_\alpha \tag{A.8}$$

$$G(r''')E_\alpha = G(t_2)G(r)E_\alpha \in \{1,-1\}E_\alpha \tag{A.9}$$

As was summarized in Tables 4 and 5, we can use these assignments to break E_6 to either $SO(10)\times U(1)_\chi$, $SU(6)\times SU(2)_L$, $SU(6)\times SU(2)_R$ or the intersection of either pair, $SU(4)\times SU(2)_L\times SU(2)_R\times U(1)_\chi$. The fixed points themselves always exhibit the local gauge invariance E_6, $SO(10)\times U(1)_\chi$ or $SU(6)\times SU(2)$.

A.3.2 Trinification on a $\mathbb{Z}_3$ Orbifold

We can obtain the trinification group $SU(3)_C\times SU(3)_L\times SU(3)_R$ by modding out the torus, $T^2/\mathbb{Z}_3$, corresponding to the geometry $\mathbb{R}^2/\mathbf{333}$ as shown in Fig. 6. The fundamental space is a 3-pillow. It is generated by a 120° rotation r about the origin, which we can give a $\mathbb{Z}_3$ shift embedding,

$$G(r)E_\alpha \in \{1, e^{2\pi i/3}, e^{4\pi i/3}\}E_\alpha\ . \tag{A.10}$$

In addition, we can assign discrete Wilson lines to the translations which span the torus, which induce different gauge shifts for the two rotations $r' = t_1 t_2 r$ and $r'' = t_1 r$,

$$G(t_1)E_\alpha = G(t_2)E_\alpha \equiv G(t)E_\alpha \in \{1, e^{2\pi i/3}, e^{4\pi i/3}\}E_\alpha \tag{A.11}$$

$$G(r') = G(t)^2 G(r), \quad G(r'') = G(t)G(r) = G(t)^{-1}G(r') \tag{A.12}$$

This gives us the possibility to have reduced gauge symmetry $SU(3)^3$ on none, two or all three corners of the pillow. We can choose for example

$$G(r)E_\alpha = \exp\left[2\pi i\,(0,0,\frac{1}{3},-\frac{1}{3},0,0)\cdot\Delta(\alpha)\right]E_\alpha, \quad G(t)=1 \tag{A.13}$$

as given in Table 4. The resulting invariance $SU(3)^3$ contains the $G_{LR}\times U(1)_\chi$ subgroup which can for example be obtained if an additional vev in the direction t_{B-L}

is present. Since all fixed points are invariant under $SU(3)^3$ only, it is interesting to consider what is the most general invariant superpotential for each. Inspired by the anomaly-free E_6 particle content, we start with the decomposition of the **27** under the trinification group, which is

$$\mathbf{27} \to \underbrace{(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3})}_{\mathbf{A}} + \underbrace{(\mathbf{3}, \mathbf{3}, \mathbf{1})}_{\mathbf{B}} + \underbrace{(\mathbf{1}, \overline{\mathbf{3}}, \overline{\mathbf{3}})}_{\mathbf{C}} \tag{A.14}$$

where **A** contains right-handed quarks q_R^c and an exotic D^c, **B** contains left-handed quarks q_L and an exotic D, while **C** contains all Higgs H_u, H_d fields including the SM singlet S, and all leptons. The E_6-invariant renormalizable trilinear superpotential decomposes into four independent $SU(3)^3$ invariants,

$$\mathbf{27^3} \to c_1\mathbf{A}^3 + c_2\mathbf{B}^3 + c_3\mathbf{C}^3 + c_4\mathbf{ABC}\ , \tag{A.15}$$

which is already the most general renormalizable superpotential one can write down for this field content. The first term $\mathbf{A}^3$ contains only diquark-like couplings to D^c, while $\mathbf{B}^3$ only contains diquark-like couplings to D. The remaining two invariants give us a complete MSSM-like superpotential including an effective μ Term SH_uH_d, leptoquark couplings and effective leptoquark masses SD^cD. If we demand $c_1 = c_2 = 0$ by some symmetry, D and D^c thus become true leptoquarks. Here, any symmetry will do under which **A** and **B** are odd while **C** is uncharged such as baryon number or parity. Even with intermediate breaking to $G_{SM} \times U(1)$ for example from an adjoint vev $\langle \mathbf{78} \rangle \sim Q_{B-L}$, none of the $SU(3)^3$ subgroups unify below the Planck scale assuming the full E_6 particle content (with right-handed neutrinos massive at the intermediate scale). Therefore, one has to invoke additional physics to achieve unification, either incomplete E_6 multiplets or extra-dimensional effects [24, 25].

A.3.3 Pati-Salam Symmetry on a $\mathbb{Z}_4$ Orbifold

The simplest 6D orbifold symmetry without reflections which can accomodate $PS \times U(1)_\chi$ fixed points is the $T^2/\mathbb{Z}_4$ orbifold $\mathbb{R}^2/\mathbf{442}$. It is shown in Fig. 7. The order 4 rotation r together with the translations t_1 and t_2 induce an order 4 and an order 2 rotation

$$r'' = t_1 r, \quad r' = t_1 r^2 \tag{A.16}$$

where $rt_1 = t_2 r$. There is one discrete Wilson line $G(t) = G(t_1) = G(t_2)$ of order two. To obtain a model with 4D gauge invariance $PS \times U(1)_\chi$ with fixed points that allow H parity and the suppression of lepto-diquark couplings, we can for example choose

$$\begin{aligned} G(r)E_\alpha &= \exp\left[2\pi i\,(\frac{3}{4},\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{1}{4},0)\cdot\Delta(\alpha)\right]E_\alpha \\ G(t)E_\alpha &= \exp\left[2\pi i\,(\frac{1}{2},\frac{1}{2},0,\frac{1}{2},\frac{1}{2},0)\cdot\Delta(\alpha)\right]E_\alpha \end{aligned} \tag{A.17}$$

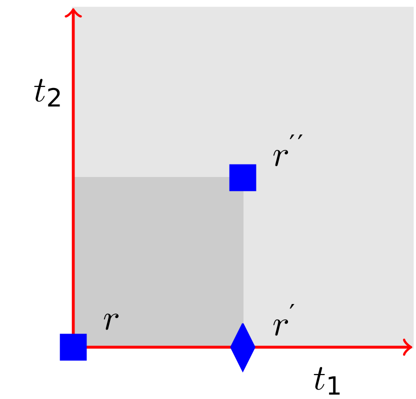

Figure 7: The $\mathbb{R}^2/\mathbf{442}$ orbifold which is a $T^2/\mathbb{Z}_4$ orbifold. The blue rectangles and and diamond indicate the fixed points under the 90° and 180° rotations respectively, while the red arrows are the translations which span the torus. The light shaded area indicates the torus, the dark shaded area corresponds to the fundamental domain of the orbifold.

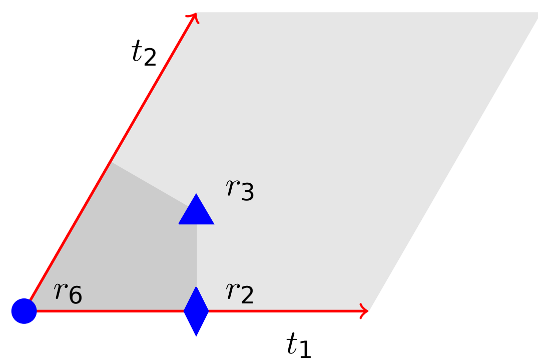

Figure 8: The $\mathbb{R}^2/\mathbf{632}$ orbifold which is one of the $T^2/\mathbb{Z}_6$ orbifolds. The blue circle, triangle and diamond indicate the fixed points under the 60°, 120° and 180° rotations respectively, while the red arrows are the translations which span the torus. The light shaded area indicates the torus, the dark shaded area corresponds to the fundamental domain of the orbifold.

as given in Table 4. Now, E_6 is broken down to $PS \times U(1)_\chi$ at the order 4 fixed points and remains unbroken at the order 2 fixed point. If one chooses instead $G(t) = 1$, it is broken to $SO(10) \times U(1)_\chi$ at the order 2 fixed point, allowing H parity.

A.3.4 Minimal LR Symmetry on a $\mathbb{Z}_6$ Orbifold

The simplest 6D orbifold geometry which can contain both nontrivial $\mathbb{Z}_2$ and $\mathbb{Z}_3$ shifts is the $T^2/\mathbb{Z}_6$ orbifold $\mathbb{R}^2/\mathbf{632}$ or p6. It is shown in Fig. 8. The order 6 rotation r_6 together with the translations t_1 and t_2 induces an order 2 and an order 3 rotation

$$r_2 = t_1 r_6^3, \quad r_3 = t_1 r_6^2 . \tag{A.18}$$

Although we are not allowed to assign discrete Wilson lines since from equation (A.18) follows that $G(t_1)^2 = G(t_1)^3 = \mathbb{1}$, we can choose gauge shifts for the $\mathbb{Z}_2$ and $\mathbb{Z}_3$ ro-

tation separately. The possible shifts satisfy

$$G(r_2)E_\alpha = G(r_6)^3 E_\alpha \in \{1, -1\} E_\alpha \tag{A.19}$$

$$G(r_3)E_\alpha = G(r_6)^2 E_\alpha \in \{1, e^{2\pi i/3}, e^{4\pi i/3}\} E_\alpha \ , \tag{A.20}$$

and vice versa

$$G(r_6)E_\alpha = G(r_2)G(r_3)^{-1} E_\alpha \in \{1, e^{\pi i/3}, e^{2\pi i/3}, -1, e^{4\pi i/3}, e^{5\pi i/3}\} E_\alpha \ . \tag{A.21}$$

B. Conventions

B.1 Dirac Algebra

Using $\sigma^0 = \overline{\sigma}^0 = 1$, $\sigma^i = -\overline{\sigma}^i$ and the 5D Dirac algebra

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \tag{B.1}$$

we define the 6D Dirac algebra as

$$\Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \gamma^\mu & 0 \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & \gamma^5 \\ \gamma^5 & 0 \end{pmatrix}, \quad \Gamma^6 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{B.2}$$

and the 6D chirality operator as

$$i\Gamma^7 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{B.3}$$

In this basis,

$$\exp\left[\frac{\phi}{4}\left[\Gamma^5, \Gamma^6\right]\right] = \mathrm{diag}\left(e^{-i\phi/2}, e^{i\phi/2}, e^{i\phi/2}, e^{-i\phi/2}\right) \sim U(1) \subset SO(1,5) \tag{B.4}$$

corresponds to a counter-clockwise rotation with angle ϕ about the origin in the extra dimensional space,

$$\overline{\Psi}(\Gamma^5 + i\Gamma^6)\Psi \longrightarrow e^{i\phi}\,\overline{\Psi}(\Gamma^5 + i\Gamma^6)\Psi.$$

B.2 Lie Algebras

There are several ways to specify a weight vector [27]. Let

$$(\alpha^a)_k \tag{B.5}$$

$(a = 1, \ldots, 6)$ be the component vectors of the simple roots of E_6 and

$$A^{ij} = \frac{2\alpha^i \cdot \alpha^j}{|\alpha^j|^2} \tag{B.6}$$

the (symmetric) Cartan Matrix of E_6. Then, there are

1. The weight vector in the canonical basis, $\mu_k = c^i(\alpha^i)_k$
2. The Dynkin coefficients Δ where $\Delta^a(A^{-1})^{ab}(\alpha^b)_i = \mu_i$
3. The dual weights $\overline{\mu}$ where $\Delta^i = A^{ji}\frac{2}{|\alpha^j|^2}\overline{\mu}^j$

These are defined such that the scalar product of two weights in the canonical basis can be recast as

$$\mu \cdot \lambda = \Delta_\mu \cdot \overline{\lambda} = \Delta_\lambda \cdot \overline{\mu} \tag{B.7}$$

In this paper we give shifts and charges as Dynkin coefficients and in the dual basis.

B.3 E_6 Representations

The Tables 6 and 7 list the weights(roots) of the **27** and **78** representation of E_6 and the particle assignments as used in this paper. The basis choice is not unique. We define the Cartan generators in the dual basis as follows:

$$\overline{I}_L^3 = (-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \quad 0) \tag{B.8}$$

$$\overline{I}_R^{3c} = (\quad 0, \quad 0, -\frac{1}{2}, \quad 0, \quad 0, \quad 0) \tag{B.9}$$

$$\overline{Q}_e = (-\frac{1}{3}, -\frac{2}{3}, -\frac{4}{3}, -1, -\frac{2}{3}, \quad 0) \tag{B.10}$$

$$\overline{Q}_Y = (\ \frac{1}{6}, -\frac{1}{6}, -\frac{5}{6}, -\frac{1}{2}, -\frac{1}{6}, \quad 0) \tag{B.11}$$

$$\overline{Q}_{B-L} = (\ \frac{1}{6}, -\frac{1}{6}, -\frac{1}{3}, -\frac{1}{2}, -\frac{1}{6}, \quad 0) \tag{B.12}$$

$$\overline{Q}_\chi = (\ \frac{1}{2}, -\frac{1}{2}, \quad 0, \quad \frac{1}{2}, -\frac{1}{2}, \quad 0) \tag{B.13}$$

These vectors must be multiplied with the Dynkin coefficients of a state $|\mu\rangle$ to obtain the corresponding physical charges,

$$\overline{Q}_i \cdot \Delta(\mu) = Q_i(\mu)$$

In this normalization,

$$\overline{Q}_Y = \overline{Q}_{B-L} + \overline{I}_R^{3c}, \quad \overline{Q}_e = \overline{Q}_Y + \overline{I}_L^3 \tag{B.14}$$

#	**Δ weight**						I_L	I_R^c	Q_e	Q_Y	Q_{B-L}	Q_χ	**assignment**
1	1	0	0	0	0	0	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$	d_L
2	−1	1	0	0	0	0	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	−1	D
3	0	−1	1	0	0	0	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{2}$	u_R^c
4	0	0	−1	1	0	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{2}$	d_R^c
5	0	0	0	1	0	−1	$-\frac{1}{2}$	0	−1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	e_L
6	0	0	0	−1	1	1	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	−1	D^c
7	0	0	1	−1	1	−1	$-\frac{1}{2}$	$-\frac{1}{2}$	−1	$-\frac{1}{2}$	0	−1	H_d
8	0	0	0	0	−1	1	$\frac{1}{2}$	0	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$	u_L
9	0	0	1	0	−1	−1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{2}$	u_R^c
10	0	1	−1	0	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	−1	H_u
11	0	1	−1	1	−1	0	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{2}$	d_R^c
12	1	−1	0	0	1	0	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$	d_L
13	0	1	0	−1	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	−1	D^c
14	1	−1	0	1	−1	0	0	0	0	0	0	2	S
15	−1	0	0	0	1	0	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	−1	D
16	1	−1	1	−1	0	0	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	ν_R^c
17	−1	0	0	1	−1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	ν_L
18	1	0	−1	0	0	1	0	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{2}$	e_R^c
19	−1	0	1	−1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	−1	H_d
20	1	0	0	0	0	−1	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$	d_L
21	−1	1	−1	0	0	1	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	0	−1	H_u
22	−1	1	0	0	0	−1	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	−1	D
23	0	−1	0	0	0	1	$\frac{1}{2}$	0	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$	u_L
24	0	−1	1	0	0	−1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{2}$	u_R^c
25	0	0	−1	1	0	0	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{2}$	d_R^c
26	0	0	0	−1	1	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	−1	D^c
27	0	0	0	0	−1	0	$\frac{1}{2}$	0	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$	u_L

Table 6: The Dynkin coefficients and particle assignments of the weights in the fundamental representation **27** of E_6 and their quantum numbers.

#	Δ **root**						I_L	I_R^c	Q_e	Q_Y	Q_{B-L}	Q_χ	**assignment**
1	0	0	0	0	0	1	0	0	0	0	0	0	**G**
2	0	0	1	0	0	−1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{4}{3}$	$-\frac{5}{6}$	$-\frac{1}{3}$	0	X
3	0	1	−1	1	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$	$-\frac{1}{3}$	0	Q_{45}
4	0	1	0	−1	1	0	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{3}{2}$	Q_{16}
5	1	−1	0	1	0	0	$-\frac{1}{2}$	0	$-\frac{2}{3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{3}{2}$	Q_{16}
6	0	1	0	0	−1	0	0	0	0	0	0	0	**G**
7	1	−1	1	−1	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$	0	Q_{45}
8	−1	0	0	1	0	0	0	0	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	U_{45}
9	1	−1	1	0	−1	0	0	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{3}{2}$	D_{16}
10	1	0	−1	0	1	1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	0	X
11	−1	0	1	−1	1	0	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$-\frac{3}{2}$	U_{16}
12	1	0	0	0	1	−1	−1	0	−1	0	0	0	$\mathbf{W_L^-}$
13	1	0	−1	1	−1	1	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{3}{2}$	U_{16}
14	−1	0	1	0	−1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{5}{6}$	$-\frac{1}{3}$	0	X
15	−1	1	−1	0	1	1	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{3}{2}$	D_{16}
16	1	0	0	1	−1	−1	$-\frac{1}{2}$	0	$-\frac{2}{3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{3}{2}$	Q_{16}
17	1	0	0	−1	0	1	0	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	U_{45}
18	−1	1	0	0	1	−1	$-\frac{1}{2}$	0	−1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	L_{16}
19	−1	1	−1	1	−1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{6}$	$-\frac{1}{3}$	0	Q_{45}
20	0	−1	0	0	1	1	0	0	0	0	0	0	**G**
21	1	0	1	−1	0	−1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$	0	Q_{45}
22	−1	1	0	1	−1	−1	0	0	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	U_{45}
23	−1	1	0	−1	0	1	$\frac{1}{2}$	0	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{3}{2}$	Q_{16}
24	0	−1	1	0	1	−1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{4}{3}$	$-\frac{5}{6}$	$-\frac{1}{3}$	0	X
25	0	−1	0	1	−1	1	$\frac{1}{2}$	0	$\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{3}{2}$	Q_{16}
26	1	1	−1	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	0	X
27	−1	1	1	−1	0	−1	0	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$-\frac{3}{2}$	U_{16}
28	0	−1	1	1	−1	−1	0	$-\frac{1}{2}$	−1	−1	$-\frac{1}{2}$	$\frac{3}{2}$	E_{16}
29	0	−1	1	−1	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$	0	Q_{45}
30	0	0	−1	1	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$	$-\frac{1}{3}$	0	Q_{45}
31	2	−1	0	0	0	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	L_{16}
32	−1	2	−1	0	0	0	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{3}{2}$	D_{16}
33	0	−1	2	−1	0	−1	0	−1	−1	−1	0	0	$E_{45} \equiv \mathbf{W_R^-}$
34	0	0	−1	2	−1	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{3}{2}$	ν_{16}
35	0	0	0	−1	2	0	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{3}{2}$	Q_{16}
36	0	0	−1	0	0	2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	0	X

Table 7: The positive roots of E_6, their Dynkin coefficients and quantum numbers. The naming scheme is with respect to the $E_6 \to SO(10)\times U(1)_\chi$ branching $\mathbf{78} \to \mathbf{45_0}+\mathbf{16_{-3/2}}+\overline{\mathbf{16}}_{\mathbf{3/2}}+\mathbf{1_0}$ as it is relevant to this paper. The six zero roots are not shown, the elements of the G_{LR} algebra are highlighted.

C. Matter and Gauge Multiplets on the $T^2/\mathbb{Z}_n$ orbifold

C.1 Gauge Theory

Upon compactification, the 6D Lorentz group is broken to

$$SO(1,5) \to SO(1,3) \times U(1) \tag{C.1}$$

so the 6D vector is split into a 4D vector A_μ and a scalar in the adjoint of the gauge group,

$$\Sigma = \frac{1}{\sqrt{2}}(A_6 + iA_5). \tag{C.2}$$

The latter transform nontrivially under the $\mathbb{Z}_n \subset U(1)$ of the orbifold space group. In particular, under $\mathbb{Z}_n$ rotations $x_5 - ix_6 \to e^{-2\pi i/n}(x_5 - ix_6)$,

$$\Sigma \to e^{-2\pi i/n}\Sigma \tag{C.3}$$

If there is an additional gauge shift associated with the orbifold action, the vector components belonging to a root E_α transform as

$$A_\mu^\alpha \to e^{2\pi i V\cdot\alpha} A_\mu^\alpha, \quad \Sigma^\alpha \to e^{2\pi i V\cdot\alpha} e^{-2\pi i/n}\Sigma^\alpha \tag{C.4}$$

Massless modes have constant Kaluza-Klein wavefunctions in the extra dimension, and therefore only appear for components which are invariant under r_6. This means that the unbroken gauge group is determined by the set of roots α which satisfy $V \cdot \alpha \in \mathbb{Z}$, which naturally includes the zero weights and thus rank is preserved. The zero modes of scalars correspond to roots which are broken on all fixed points. In that sense the situation is similar to a $S^1/\mathbb{Z}_2$ orbifold where massless scalars appear if a generator is broken on both ends of the fundamental domain.

C.2 Supersymmetric theory

Hypermultiplets coupled to supersymmetric Yang-Mills theory in 5 and 6 dimensions can be formulated using $\mathcal{N} = 1$ superfields [26]. The field content is identical in both cases, namely that of 4D $\mathcal{N} = 2$ supersymmetry. In this language, the gauge hypermultiplet consists of a vector superfield $\hat{V}$ and a chiral superfield $\hat{\chi}$ in the adjoint. The physical scalar component of the latter contains the extra dimensional gauge field components, $\hat{\chi}| = \Sigma + \mathcal{O}(\theta)$. There are now three real auxiliary fields forming a triplet under the R symmetry, $SU(2)^R$. The hypermultiplet can be written as two chiral superfields with opposite charge, $\hat{\Phi}$ and $\hat{\Phi}_c$. The chiral fermionic SUSY parameters can be defined as a 6D spinor $\xi = (\xi_1, \overline{\xi}_2, 0, 0)^T$. In general, these parameters transform nontrivially under orbifold rotations. Given the 6D Dirac matrices, Γ^M, the $\mathbb{Z}_n$ rotations in the 5-6 plane of orbifolds on T^2 are generated by

$$\theta = \exp\left[\frac{2\pi}{n}\frac{1}{4}[\Gamma^5, \Gamma^6]\right].$$

With trivial embedding in the R symmetry, the SUSY parameter transforms as

$$\begin{pmatrix} \xi_1 \\ \overline{\xi}_2 \end{pmatrix} \xrightarrow{\theta} \begin{pmatrix} e^{-i\pi/n}\xi_1 \\ e^{i\pi/n}\overline{\xi}_2 \end{pmatrix} \tag{C.5}$$

which would result in a non-supersymmetric massless spectrum, and $\theta^n \neq 1$.

It is known from 10D orbifold constructions (such as heterotic models), that certain conditions have to be fulfilled in order to preserve at least $\mathcal{N}=1$ SUSY in 4D[6]. The smallest nontrivial spinor in 10D is both chiral and real, and thus corresponds to 4D $\mathcal{N}=4$ SUSY. General $\mathbb{Z}_n$ transformations on the complex torus coordinates are generated by

$$(z_A, z_B, z_C) \to (e^{iA}z_A, e^{iB}z_B, e^{iC}z_C)\,. \tag{C.6}$$

Using a 10D dirac algebra, the spinors then transform as

$$\theta = \exp\left[\frac{A}{4}[\Gamma^5,\Gamma^6] + \frac{B}{4}[\Gamma^7,\Gamma^8] + \frac{C}{4}[\Gamma^9,\Gamma^{10}]\right] \tag{C.7}$$

The condition that we want to preserve at least one 4D supersymmetry can now be formulated as a constraint on the coefficients,

$$A + B + C = 0 \quad , \tag{C.8}$$

which is not possible in 6D, where $B = C \equiv 0$. However, after trivial dimensional reduction to 6 dimensions, the $SO(4) \simeq SU(2)\times SU(2)'$ Lorentz algebra of the higher dimensions generated by $\Gamma^7 \ldots \Gamma^{10}$ becomes an internal symmetry. It contains the $SU(2)^R$ under which the chiral 6D $\mathcal{N}=1$ SUSY generator is a doublet. Thus, we have the choice to either introduce additional (possibly small) spacetime dimensions with nontrivial rotational phases that cancel the chiral phases to preserve 4D $\mathcal{N}=1$ supersymmetry (this however might be viable only in the context of a heterotic string theory). Or we can start with a 6D $\mathcal{N}=1$ setup and assign an embedding of $\mathbb{Z}_n$ in I^{3R} of $SU(2)^R$ to the chiral 6D spinors,

$$\theta = \exp\left[\frac{2\pi}{n}\frac{1}{4}([\Gamma^5,\Gamma^6] + c_R i I^{3R})\right]$$

where the constant c_R can be chosen appropriately to conserve at least one 4D supersymmetry, and $\theta^n = 1$. From now on we assume that the conserved 4D $\mathcal{N}=1$ supersymmetry is chosen such that it acts as usual within the $\mathcal{N}=1$ superfields $\hat{V}, \hat{\chi}, \hat{\Phi}, \hat{\Phi}^c$. For the orbifold construction this means that orbifold phases are assigned to entire superfields, and the resulting massless multiplets are given directly as zero modes of complete $\mathcal{N}=1$ superfields. The transformations in (C.4) are generalized to

$$\hat{V}^\alpha \to e^{2\pi i V\cdot\alpha}\hat{V}^\alpha, \quad \hat{\chi}^\alpha \to e^{2\pi i V\cdot\alpha}e^{-2\pi i/n}\hat{\chi}^\alpha \tag{C.9}$$

[6]We thank P. Vaudrevange for clarifications concerning this problem.

Then, zero modes of A_μ automatically come with complete 4D vector multiplets, while zero modes of A_5 and A_6 come with complete 4D chiral multiplets. The discussion of ordinary YM theory can therefore be generalized to the SYM case in a straightforward manner.

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